

Repeated limit If a function  $f$  is defined in some neighbourhood of  $(a, b)$  then the limit

$$\lim_{y \rightarrow b} f(x, y)$$

If it exists, is a function of  $x$  say  $\phi(x)$ . If then the limit  $\lim_{x \rightarrow a} \phi(x)$  exists and is equal to  $\lambda$

we write

$$\lim_{x \rightarrow a} \lim_{y \rightarrow b} f(x, y) = \lambda$$

and say that  $\lambda$  is a repeated limit of  $f$  as  $y \rightarrow b, x \rightarrow a$

If we change the order of taking the limits, we get the other repeated limit

$$\lim_{y \rightarrow b} \lim_{x \rightarrow a} f(x, y) = \lambda' \text{ (say)}$$

when first  $x \rightarrow a$  and then  $y \rightarrow b$

These two limits may or may not be equal.

Example (1) Let

$$f(x, y) = \frac{xy}{x^2 + y^2} \text{ then}$$

$$\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y) = \lim_{x \rightarrow 0} (0) = 0$$

$$\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y) = 0$$

Thus the repeated limits exist and are equal. But the simultaneous limit does not exist which may be seen by putting  $y = mx$ .